Experimentally Determined Impedance Characteristics of Cylindrical Antennas*

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Summary—Measurements of resistance and reactance of cylindrical antennas operated against ground have been made, with a wide variation of both antenna length and diameter. These data are displayed by means of a series of graphs.

The maximum values of resistance encountered are displayed. The shortening effect near the quarter-wave resonance point is also shown.

Terminal conditions, such as capacitance of the base of the antenna to ground, are briefly considered, and a series of measurements shows the wide variation in impedance for varying terminal conditions.

Measurements made in the course of the investigation show that the impedance of the antenna is independent of whether the top of the radiator is open or closed. The measured impedance data are also directly applicable to the case of a center-fed dipole.

I. INTRODUCTION

A KNOWLEDGE of the base impedance of vertical antennas as a function of antenna length and diameter is very helpful in devising terminating networks for antenna systems. Much has been written concerning the mathematical difficulties of rigorously solving the antenna problem, and several methods of approximation have been proposed. Very little information of an experimental nature has been published. A number of years ago, the writers decided to undertake a systematic investigation of the problem. Other projects of a more pressing nature have seriously impaired our plans. However, the work of measuring the resistance and reactance of simple cylinders has been completed and the data compiled. The purpose of this paper is to present these data in a form that may prove useful.

II. METHOD OF MEASUREMENT

The physical arrangement for making the measurements is shown in Fig. 1. A large circular metallic screen 12 feet in diameter and 15 wires to the inch was placed on the surface of the earth. A concentric transmission line ran below this screen back to a slotted section of measuring line. Here a sensitive probe indicated the ratio of minimum-to-maximum voltage on the line as well as the position of a voltage minimum on the line. These two quantities together with the characteristic impedance of the feed line establish the impedance that exists at the end of the line. The inner diameter of the outer conductor of the feed line was 0.785 inch, while the diameter of the inner conductor was 0.25 inch. The cylinder which formed the antenna was closed at the bottom with a circular metal plate.

The writers rather like the method of notation which expresses dimensions in electrical degrees. For instance, in Fig. 1, the dimension a which is the physical length from the ground plane to the top of the antenna may be used to compute the length A in electrical degrees.

$$A(\text{degrees}) = 360a/\lambda \qquad (1)$$

where λ is the free-space wavelength measured in the same units in which a is measured. The actual diameter d may be used in the same way to express the diameter D in electrical degrees.

Two simple and useful formulas for computing A are

$$A(\text{degrees}) = a_{\text{ft}}f_{kc}/2725 \qquad (2)$$

and

$$A(\text{degrees}) = a_{\text{inches}}f_{mc}/32.7. \qquad (3)$$

Measurements were made on wires and cylinders of various diameters, with D ranging from 0.1 degree to 20 degrees. This range was chosen with an eye to practical considerations, for a 3-inch diameter mast at 1 megacycle represents a value of D close to 0.1 degree, while a diameter of 1.3 inches at 500 megacycles is approximately 20 degrees.

The spacing h between the ground plane and the metal plate closing the bottom of the cylinder was chosen so that this electrical spacing H was one degree.

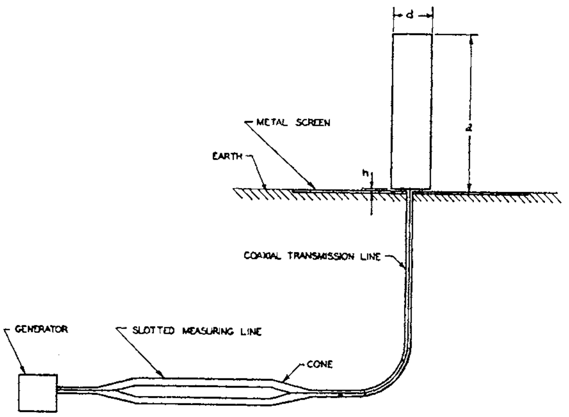

Fig. 1—Physical arrangement used in making impedance measurements.

III. RESISTANCE AND REACTANCE VARIATION, WITH DIAMETER AND FREQUENCY CONSTANT, AND THE ANTENNA LENGTH VARIABLE

A complete series of measurements was made by choosing a cylinder of a certain diameter, maintaining a fixed frequency of 60 megacycles, while the physical length of the antenna was changed. The resulting resistance curves are shown in Fig. 2, while the corresponding reactance values are presented in Fig. 3.

From Fig. 2, we see that the maximum value of

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* Decimal classification: R241.1×R320. Original manuscript received by the Institute, September 18, 1944; revised manuscript received, November 30, 1944.

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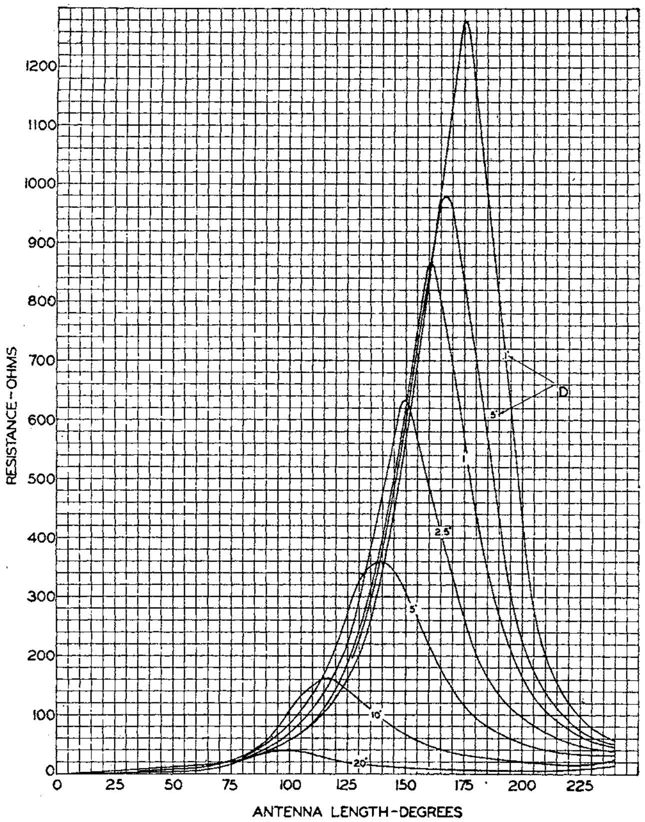


Fig. 2—Measured resistance curves versus antenna length in degrees, for a number of fixed diameters. Here the frequency is held constant and the physical length of the antenna is changed.

resistance for each diameter becomes greater and occurs closer to $A=180$ degrees as the diameter becomes smaller. This point is illustrated strikingly by Fig. 4. This diagram was constructed from Fig. 2. The top curve in Fig. 4 shows the antenna length A at which maximum resistance occurs for each value of D. The corresponding value of the maximum resistance is shown by the lower curve. The upper curve extrapolates very nicely and shows that as D approaches zero, for maximum resistance, A approaches 180 degrees. The lower curve shows that there is a good chance of the resistance approaching infinity as the diameter approaches zero.

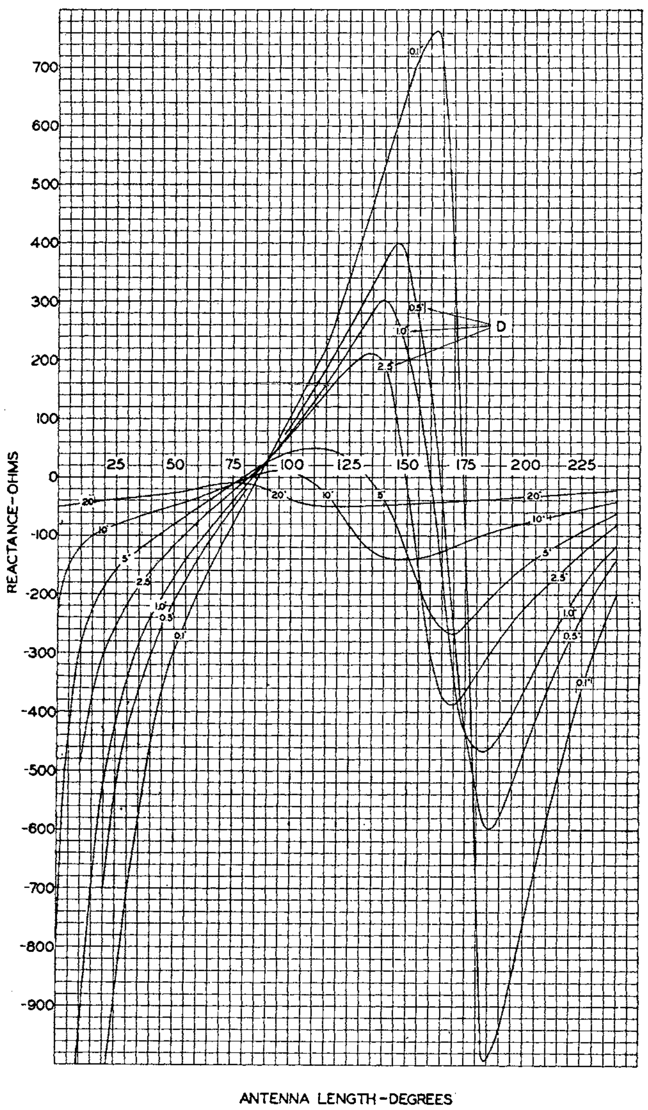


Fig. 3—Measured reactance curves versus antenna length in degrees. Again the frequency is held constant and the physical length of the antenna is changed.

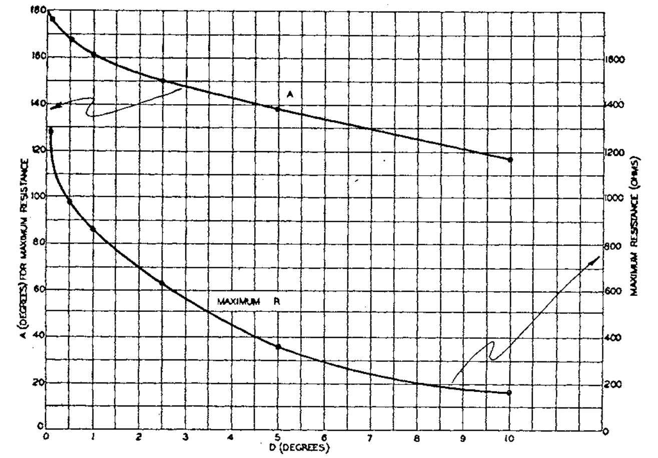


Fig. 4—Maximum resistance versus antenna diameter D and the antenna length A at which the maximum resistance occurs versus antenna diameter.

Fig. 3 shows that as the antenna becomes very short, the reactance approaches a definite limiting value. The effect is particularly striking in the case where D is 20 degrees. Actually, in our measurements, the antenna length did not go down to zero degrees, for when the antenna has been trimmed entirely away, the plate which formed the bottom of the cylinder still remained, so that we measured the reactance of this disk hung on the end of the measuring line. It should be realized that all of our measurements shown in Figs. 2 and 3 are the combined impedances of the antenna proper in parallel with the base reactance. Theoreticians may well object that this prevents comparison between theoretical and experimental results. It is entirely possible to imagine a cylinder with the base removed and excited by a number of tiny generators connected between the periphery and the ground plane. Then a knowledge of the relationship between the generator voltage and the current delivered by the generators would give an answer closer to the ideal, since the base charging current would not be present. However, a realistic consideration of the problem soon indicates that these measurements taken under true existing conditions are likely to be of more value from a design standpoint.

It might be added that the effect of this base capacitance only makes itself felt where *D* is larger than one degree.

It may be observed from Figs. 2 and 3, that the reactance curve crosses through zero close to the point where the resistance is maximum. There is, however, some slight departure from true correspondence. The solid curve in Fig. 5 shows the antenna length for zero reactance as a function of the diameter, *D*. By calculation from Figs. 2 and 3, the writers determined the zero reactance point when the base reactance was removed. The resulting values of antenna length are shown by the broken curve of Fig. 5.

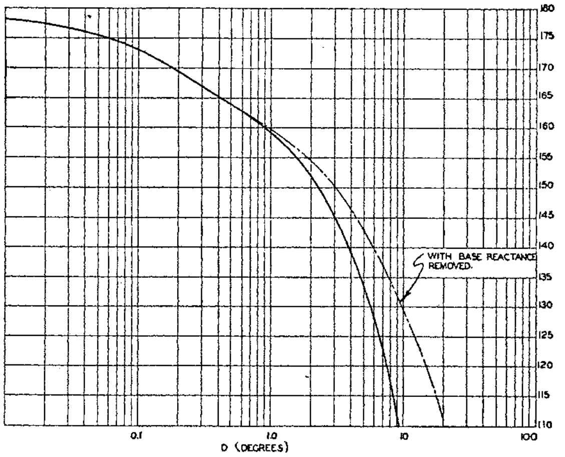


Fig. 5—The solid curve shows the antenna length *A* for zero reactance (near the maximum resistance point) as a function of the diameter *D*. This information is taken from measured data. The broken curve shows the same quantities when the base reactance is removed by calculation.

IV. RESISTANCE AND REACTANCE VARIATION, WITH DIAMETER AND LENGTH CONSTANT, AND WITH VARIABLE FREQUENCY

The curves of Figs. 2 and 3 show the impedance variation when the antenna length is varied. Often, it is of interest to know the action for a fixed antenna as the frequency is varied. By working through the curves of Figs. 2 and 3, and by cross-plotting much of the experimental data, the writers were able to build up Figs. 6 and 7. Here the resistance and reactance variation is shown for a fixed ratio of antenna length to diameter, and the variation of the electrical length of the antenna is secured by varying the frequency.

It may be noted that the reactance curves approach infinite values as the antenna length approaches zero, since the approach to zero antenna length is secured by approaching zero frequency.

Figs. 6 and 7 may prove to be useful in designing antennas to cover a wide frequency range.

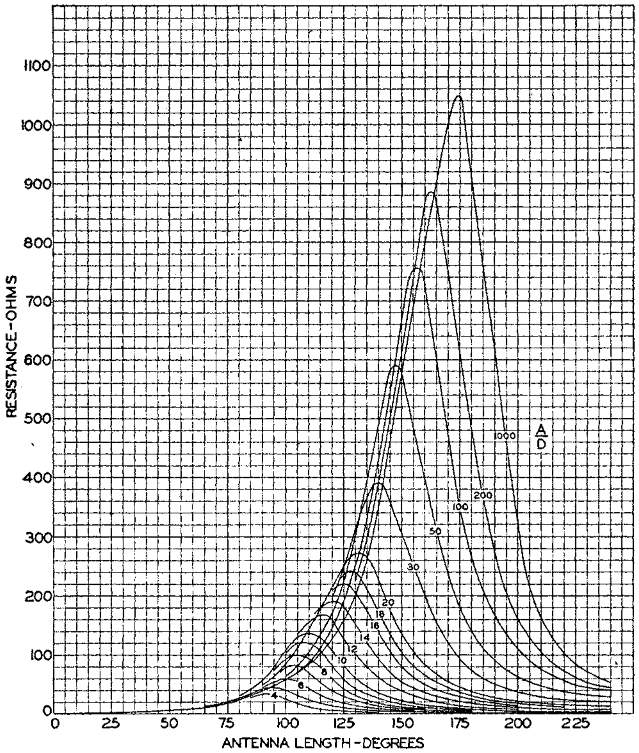


Fig. 6—Antenna resistance versus antenna length *A*, when a constant ratio of length to diameter *A*/*D* is maintained. Here the length and diameter are held constant while the frequency is changed.

Fig. 8 presents the data of Fig. 6 in a somewhat different fashion. Here each curve shows the antenna resistance as a function of the ratio of antenna length to diameter (*A*/*D*) for a fixed antenna length. Since the antenna length is fixed for each curve, large values of *A*/*D* represent very thin wires, while small values of *A*/*D* correspond to fat antennas. It is interesting to note that the resistance of a wire whose length *A* equals 90 degrees and whose *A*/*D* value is greater than 100 is very close to the theoretical value of 36.6 ohms obtained from the assumption of a simple sine-wave distribution of current.

Reactance curves as a function of *A*/*D* are given by Fig. 9. Many of these curves would be altered remarkably by removing the shunting reactance at the base, or by altering the terminal conditions.

Reference to Fig. 6 helped in the preparation of Fig. 10, where the maximum resistance as a function of *A*/*D* is shown.

Another interesting bit of information may be extracted from the data shown in Fig. 7 and Fig. 9. It is generally known that the first resonance in a vertical antenna occurs close to a length of *A* equal to 90 degrees, and it is also general knowledge that the antenna should be shortened slightly from the 90-degree length to obtain this resonance or zero-reactance condition. By cross-plotting the data of Figs. 7 and 9, the writers obtained Fig. 11, which shows the shortening (expressed in per cent of 90 degrees or one-quarter wavelength) necessary to secure zero reactance for each value of *A*/*D*.

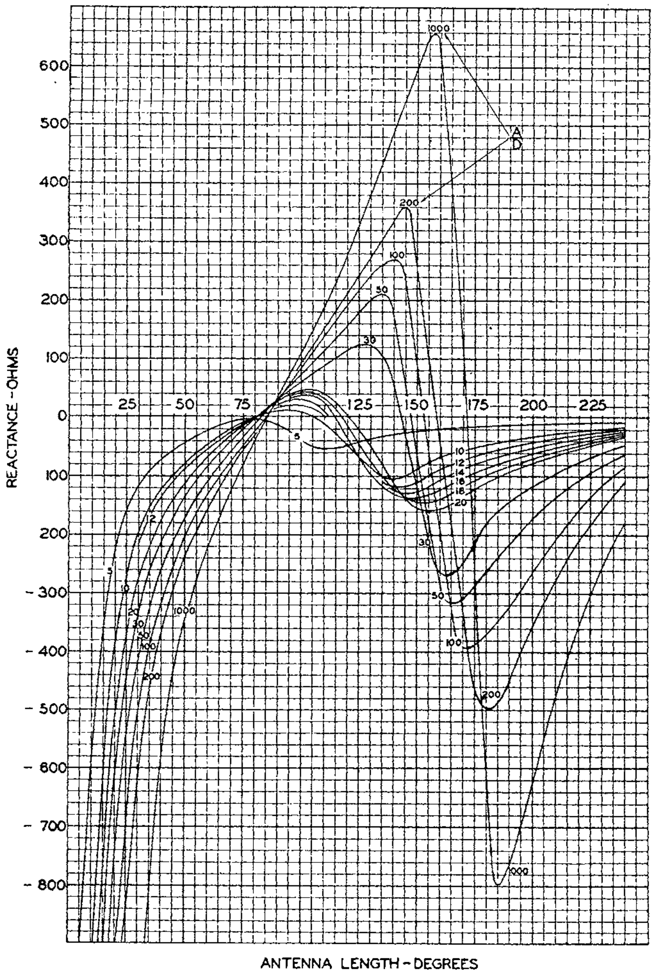


Fig. 7—Reactance curves corresponding to the resistance curves of Fig. 6.

V. REACTANCE OF THE BASE-PLATE

As has been stated, the disk which closes off the bottom of the radiator forms a shunt capacity across the terminals of the radiator. We may estimate the amount of the base shunting reactance by calculating the capacitance of the disk, neglecting fringing at the edges, and assuming that all displacement currents flow from the bottom of the disk to the ground plane. Then the shunt reactance is

$$X_s = \frac{\mu c^2 h}{\omega(\pi d^2/4)} \tag{4}$$

where μ = permeability of free space $= 4\pi \times 10^{-9}$

c = velocity of radio waves in free space $= 3 \times 10^{10}$ centimeters per second

h = spacing of disk from plane (centimeters)

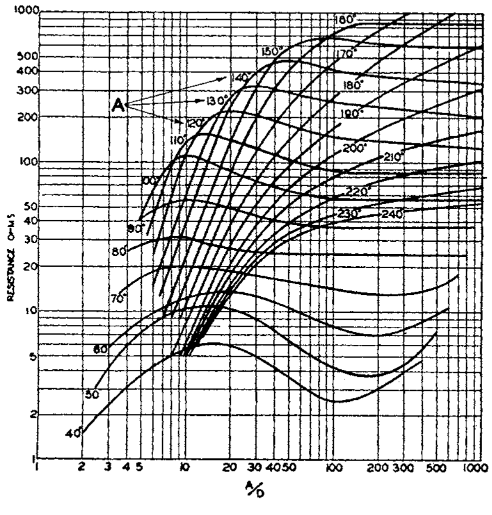


Fig. 8—Resistance versus the ratio A/D with antenna length A as a parameter.

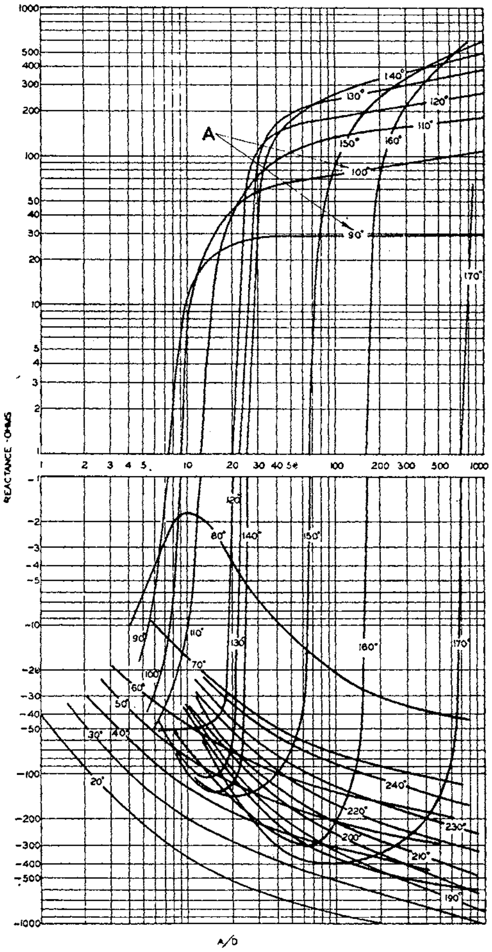


Fig. 9—Antenna reactance versus the ratio A/D for a number of values of antenna length A.

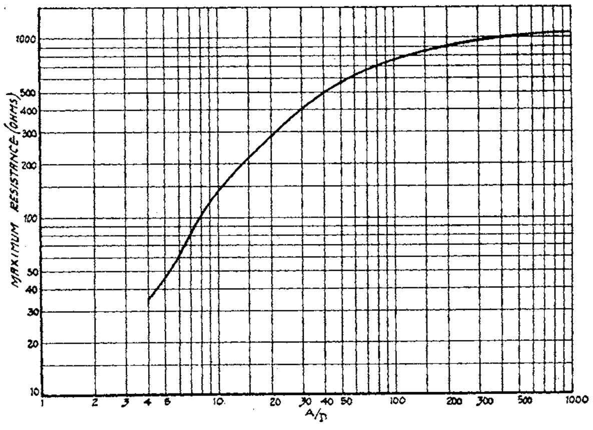


Fig. 10—Maximum resistance as a function of the ratio A/D.

d = diameter of disk (centimeters)
$\omega = 2\pi f$
f = frequency in cycles per second

Then we may rewrite (4) as

$$X_s = \frac{480(\omega h/c)}{(\omega d/c)^2}. \tag{5}$$

However, if we express the spacing in electrical degrees H and the diameter in the same way D, (5) becomes

$$X_s = 27{,}500H/D^2. \tag{6}$$

In making the measurements shown in Figs. 2 and 3, we kept H equal to 1 degree. Thus, for D equal to 20 degrees, we see that the shunt capacitive reactance is 68.3 ohms, a quantity which is not at all negligible. However, when D is 1 degree, the shunt reactance is 27,500 ohms, a rather high value compared to any value of impedance encountered during the course of measurement.

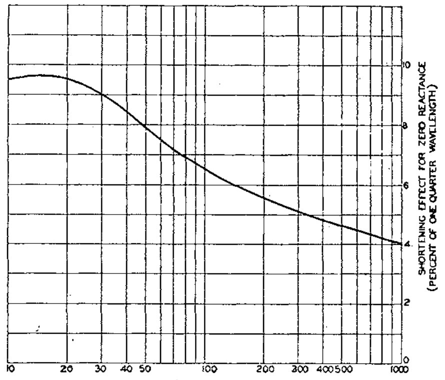
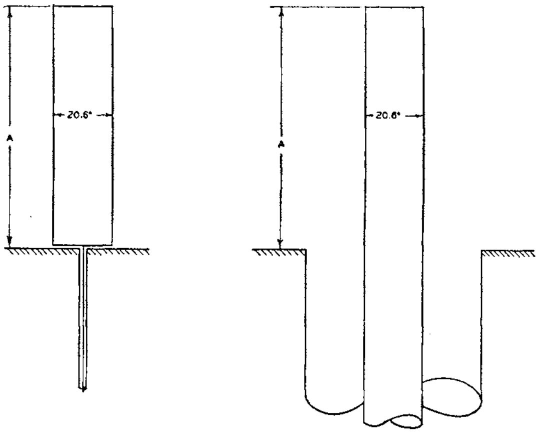


Fig. 11—Shortening effect near the quarter-wave point.

VI. COMPARISON OF IMPEDANCE MEASUREMENTS WITH VARYING TERMINAL CONDITIONS

For the previously disclosed measurements with large diameters, the experimental arrangement looked something like that shown in Fig. 12(a). Another experimental arrangement used for comparison purposes was that shown in Fig. 12(b). Here the inner conductor of the measuring line was the same diameter as the antenna. In fact, the antenna was simply the extension of the inner conductor. The system was so arranged that no insulators were in the measuring line between the point of measurement and the antenna. In these comparative measurements, the antenna diameter was maintained at 20.6 degrees. Three sizes of tubing were chosen for the outer conductor of the transmission line. The diameters of the transmission line as well as the characteristic impedance are given in the captions for Figs. 13 and 14.

Fig. 13 shows the measured resistance values for the arrangements, while Fig. 14 shows the corresponding reactance curves. Curve A in both figures shows the measured values for the arrangement of Fig. 12(a), with a diameter of 20.6 degrees. For this diameter, (6) shows that the shunt capacitive reactance is 65.0 ohms.



(a) (b)

Fig. 12—Experimental arrangement for obtaining curves in Figs. 13 and 14. (a) For curve A. (b) For curves B, C, and D.

To knock out the effect of this shunting reactance, we may imagine an inductive reactance in shunt with the antenna, where this auxiliary reactance has a value of 65.0 ohms. Curve E on Figs. 13 and 14 was computed in exactly this fashion.

For instance, with an antenna length of 100 degrees, curve A shows that R_A is 42.0 ohms and X_A is -39.5 ohms. Then to find the impedance without the shunt capacitance, we calculate the parallel circuit conditions.

$$R_E + jX_E = \frac{j65.0(42.0 - j39.5)}{42.0 + j25.5} = 74.0 + j20.5 \text{ ohms.}$$

Examination of Figs. 13 and 14 shows that curve E fits in with the group formed by curves B, C, and D, particularly with regard to reactance values. This illustrates the point that the excessive base shunting reactance materially effects the measured impedance values. The difference between curves B, C, and D may be attributed again to changing terminal conditions.

The fact that curve E does not conform better to the curves B, C, and D is probably due to the fact that the simple conditions postulated in setting up (6) do not take full account of conditions near the terminals, and

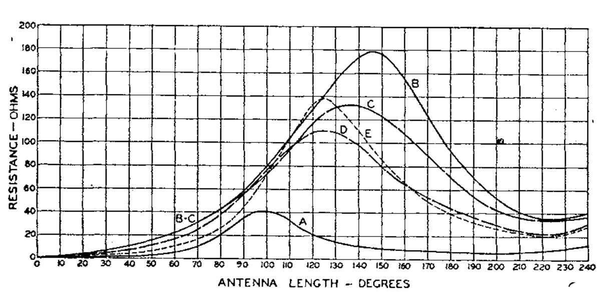


Fig. 13—Resistance as a function of antenna length A. The diameter D is 20.6 degrees.
Curve A—The arrangement shown in Fig. 12(a).
Curve B—The arrangement of Fig. 12(b), with the diameter of the outer conductor equal to 74 degrees. The characteristic impedance of the transmission line is 77.0 ohms.
Curve C—The outer conductor diameter is 49.5 degrees, and the transmission line has a characteristic impedance of 52.5 ohms.
Curve D—The diameter of the outer conductor is 33 degrees. The characteristic impedance is 28.3 ohms.
Curve E—This curve was obtained by tuning out the base reactance with an inductive reactance of 65.0 ohms.

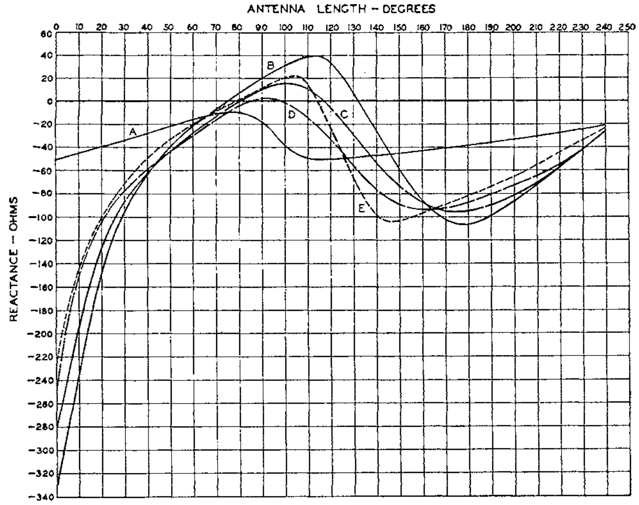


Fig. 14—Reactance curves corresponding to the resistance curves of Fig. 13.

in addition to the fact that curve A was measured at 60 megacycles, while curves B, C, and D were obtained at a frequency of 540 megacycles.

VII. Comparison of Impedance with Top of Radiator Closed and Open

Some workers in the field have suggested that results obtained with the radiator closed at the top would be different than when the radiator is open.[1] The writers continually checked this point throughout the course of the experiments. A great number of measurements were made with an open radiator, and then repeated with a disk soldered into place at the top of the radiator. Particularly careful observations were made with a radiator 20 degrees in diameter.

It was originally planned to display these experimental data. This plan was abandoned when it was discovered that opening or closing the top of the radiator made not the slightest difference in the measured impedance.

VIII. Application of the Measured Curves to Center-Fed Dipoles

All the data presented in this paper have applied to an antenna fed against ground. In case a center-fed dipole is to be considered, the antenna length A throughout this paper becomes the half length of the dipole, while all values of resistance and reactance shown in this paper must be doubled to give the proper values for a dipole. Due consideration must also be given the terminal conditions.

IX. Conclusion

Measured values of resistance and reactance of cylindrical antennas operated against ground have been displayed in a number of ways. It has been demonstrated that the exact conditions at the terminals are extremely important in determining the impedance conditions.

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[1] L. Brillouin, "The antenna problem," *Quart. Appl. Math.*, vol. I, p. 214; October, 1943.

Electronic Alternating-Current Power Regulator*

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Summary—**The object of this paper is to describe an electronic alternating-current power regulator, which is instantaneous and independent of frequency.**

The theory and design considerations governing a conventional circuit using gaseous discharge tubes are presented. The effect of the extent of voltage-limiting by the gas tubes on the degree of regulation is discussed.

A bridge-type circuit is described and its theory developed. The effect of the degree of unbalance of the bridge circuit on the degree of regulation is discussed. The application of these circuits for regulation of low power, particularly the use in electronic apparatus, is treated and performance data on both circuits are given.

Introduction

FOR certain applications there is a need for alternating-current power regulators for low power. One particular application is the supply of constant heater power for vacuum tubes in direct-current amplifiers, in which varying heater power is one of the main causes of objectionable drift.

The devices used most for stabilizing low-wattage alternating-current power derived from commercial power lines either have a time lag before compensation takes effect, are dependent upon a constant power-line frequency, or are objectionable from the viewpoint of weight and cost.

Since gaseous discharge tubes have been used successfully for regulation of direct voltage, it has been suggested that the voltage-limiting characteristics of these tubes be utilized also for regulation of alternating-current power. The literature shows proposals to use gaseous discharge tubes connected across the primary winding of a power transformer.[1,2] Such an arrangement yields a degree of regulation which may satisfy certain

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* Decimal classification: 621.375.1. Original manuscript received by the Institute, October 4, 1944; revised manuscript received, December 7, 1944.

† Brown Instrument Company, Philadelphia 44, Pennsylvania.

[1] G. F. Lampkin, "A simple A. C. voltage regulator," *Electronics*, vol. 10, pp. 30, 31, 36, 39, 40; August, 1937.

[2] M. H. Sweet, "A fluorescent lamp voltage stabilizer," *Electronics*, vol. 13, pp. 60–62; August, 1940.